Lecture 19: The "S-Curve"
(Control Rod Worth and Calibration)

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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Introduction: The Misconception of the "Gas Pedal"

In a car, if you press the gas pedal 10% further, you generally expect a proportional increase in engine output. In a nuclear reactor, a Control Rod is **not linear**.

- Moving a rod 1 inch at the **bottom** of the core might add ≈ 0 reactivity.
- Moving a rod 1 inch in the **middle** of the core might add huge amounts of reactivity.
- Moving a rod 1 inch at the **top** of the core might add ≈ 0 reactivity again.

This non-linearity creates the famous **"S-Curve"** of rod worth. Understanding this curve is vital for predicting how the reactor will respond to operator actions.

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1 The Physics: Importance and Flux

Why does the rod effectiveness depend on position? We turn to **Perturbation Theory**.

The reactivity effect (ρ) of a neutron absorber depends on two things:

1. **How many neutrons are there?** (The Flux, ϕ). If there are no neutrons to catch, the rod does nothing.
2. **How important are those neutrons?** (The Adjoint Flux, $\phi^\dagger$). In a simple bare reactor, the "importance" of a location is proportional to the flux itself ($\phi^\dagger \approx \phi$).

Therefore, the effectiveness of a control rod at a specific location z is proportional to the **square** of the flux:

$$\text{Worth} \propto \phi(z)^2 \tag{1}$$

2 Differential Rod Worth

Let's consider a bare cylindrical reactor where the flux distribution along the height (H) is a sine wave:

$$\phi(z) = \phi_{max} \sin\left(\frac{\pi z}{H}\right)$$

The **Differential Rod Worth** ($\frac{d\rho}{dz}$) tells us how much reactivity we add per inch of rod movement. Based on our squared-flux rule:

$$\frac{d\rho}{dz} = C \sin^2\left(\frac{\pi z}{H}\right) \tag{2}$$

Where C is a constant related to the rod material and geometry.

Physical Interpretation:

- **Ends ($z = 0, H$):** $\sin(0) = 0$. The flux goes to zero at the boundaries due to neutron leakage and boundary conditions. The rod absorbs nothing. Moving the rod here has almost no effect.
- **Center ($z = H/2$):** $\sin(\pi/2) = 1$. The flux is maximum. The rod is "biting" into the highest population of neutrons. Moving the rod here has the **maximum** effect.

3 Integral Rod Worth (The S-Curve)

Operators don't just care about the rate of change; they care about the **Total Reactivity** inserted if they pull the rod from the bottom (0) to a height (z). To find this, we integrate the differential worth:

$$\rho(z) = \int_0^z \frac{d\rho}{dz'}\,dz' = \int_0^z C\sin^2\left(\frac{\pi z'}{H}\right)\,dz'$$

Recall the trig identity: $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$. Integrating gives:

$$\rho(z) = \rho_{total}\left[\frac{z}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi z}{H}\right)\right] \tag{3}$$

This function plots an **"S-shape"** (Sigmoid).

3.1 The "Linear" Approximation

Notice that in the center of the core (between roughly 20% and 80% withdrawal), the S-curve is relatively straight.

- Operators like to keep controlling rods in this "linear" region to make control predictable.
- If rods are too deep (near 0) or too far out (near H), the control becomes "sluggish" (large motion required for small effect).

4 Engineering Application: Rod Banking

Because a single rod distorts the flux (ϕ) so badly, we rarely move just one. We move them in symmetric groups called **Banks** to keep the power distribution radial symmetric.

Historical Case Study: The SL-1 Accident (1961)
The importance of integral rod worth was tragically demonstrated in Idaho in 1961. The SL-1 reactor was a small Army design with a single, highly effective central control rod. During maintenance, an operator was required to lift the rod 4 inches (low worth region). Instead, he withdrew it approximately 20 inches.

- By moving into the high-worth "steep" part of the S-curve, he added enough reactivity to make the reactor **Prompt Critical**.
- The resulting steam explosion killed three operators and destroyed the reactor.
- This accident led to the modern **"Stuck Rod Criterion"**: A reactor must remain subcritical even if its most valuable rod is fully withdrawn/stuck out of the core.

5 Safety Implication: Rod Ejection Accident

The ϕ^2 dependence has a terrifying safety implication. If a control rod housing fails and the high-pressure coolant shoots the rod out of the core (Rod Ejection):

- If the rod was at the **edge** (low worth), the reactor might just SCRAM safely.
- If the rod was in the **center** (high worth), the sudden removal of that much negative reactivity can drive the reactor **Prompt Critical** instantly.

This is why there are strict "Rod Insertion Limits" (RIL). You are often not allowed to insert certain high-worth rods too deeply during full power operation, to minimize the worth of a potential ejected rod.

6 Summary

- **Non-Linearity:** Control rods are not linear dials.
- **Differential Worth:** Follows a $\sin^2(z)$ shape (peaked at center).
- **Integral Worth:** Follows an S-Curve.
- **Operation:** Rods are most effective in the middle of the core.

References

1. **Textbook:** Lamarsh, J.R. and Baratta, A.J., *Introduction to Nuclear Engineering*, 4th Edition. Section 7.3 (Control Rods).
2. **Primary Source (Accident Report):** General Electric Co., *Final Report of SL-1 Recovery Operation*, IDO-19311, U.S. Atomic Energy Commission, July 27, 1962.
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(See Section III, "Nuclear Analysis," regarding the 20-inch withdrawal vs. the 4-inch requirement).
3. Flynn, George J., **SL-1 reactor accident**, EBSCO Knowledge Advantage, 2024
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